

Name of the Student: _____

Max. Marks : 24 Marks

Time : 24 Minutes

Mark Schemes

Q1.

- (a) Relating the gravitational force to mass $\times$ acceleration with the acceleration being centripetal in any form ✓₁

$$\frac{GMm}{r^2} = (ma) = \frac{mv^2}{r} \text{ or } mr\omega^2 \text{ or } mv\omega \text{ and } r \text{ can be replaced by } R + h$$

show the working leading to answer = $\omega = \sqrt{\frac{GM}{(R+h)^3}}$ ✓₂

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(b) $\left(\omega = \sqrt{\frac{GM}{(R+h)^3}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(2.02 \times 10^7 + 6.37 \times 10^6)^3}} \right)$

✓₁ A full substitution in the equation can gain this mark even if ω is incorrectly calculated

$$= 1.45 \times 10^{-4} \text{ (rad s}^{-1}\text{)} \quad \checkmark_1$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.45 \times 10^{-4}} = 4.3(1) \times 10^4 \text{ (s)} \quad \checkmark_2$$

A final correct answer for T gains both marks

2

- (c) Launch from Z with some speed or energy argument ✓₁

✓₁ Condone ref. to equator rather than Z

At this position the satellite has the largest initial speed/kinetic energy from the Earth's rotation.

✓₂

Consider answer Y only if extremely well explained in terms of different potentials and fuel use.

2

- (d) gravitational potential energy

$$= (-)\frac{GMm}{r} \text{ OR } (-)\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 1630}{2.66 \times 10^7} \quad \checkmark_1$$

$$\text{gravitational potential energy} = -2.44 \times 10^{10} \text{ (J)} \quad \checkmark_2$$

✓₁ for the equation or substitution without the need for the negative sign

only allow ecf for $r = 6.37 \times 10^6$

giving gpe = $-1.02 \times 10^{11} \text{ J}$

✓₂ the minus is necessary for the mark
Correct answer gains both marks

2

- (e) In a higher orbit the linear speed is smaller using

$$v = \sqrt{\frac{GM}{r}} \text{ or } v \propto \frac{1}{\sqrt{r}} \checkmark_1$$

Suitable justification of $v \propto \frac{1}{\sqrt{r}}$ ✓₂

✓₂ example

$$v = r\omega = r \sqrt{\frac{GM}{(R+h)^3}} = r \sqrt{\frac{GM}{(r)^3}} = \sqrt{\frac{GM}{r}}$$

Or derivation from basics

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[10]

Q2.

(a) $\frac{GmM}{r^2} (=ma) = mr\omega^2$

OR $\frac{GmM}{r^2} = \frac{mv^2}{r} \checkmark$

The starting point must be from relating forces or accelerations – not a remembered equation produced at a later stage in the manipulation of the equation.

$$\omega^2 = \left(\frac{2\pi}{T}\right)^2$$

OR $v = \frac{s}{t} = \frac{2\pi r}{T} \checkmark$

Middle mark may be given when seen as a substitution and can be a stand-alone mark.

$$\frac{GM}{r^2} = r \frac{4\pi^2}{T^2} \text{ results in } T^2 = \left(\frac{4\pi^2}{GM}\right) r^3 \checkmark$$

Last mark is only given if accompanied with the working and must eventually be in the form with T^2 and r^3 .

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(b) $\left(\frac{T_U^2}{T_M^2} = \frac{r_U^3}{r_M^3}\right)$

$$r_U = r_M \left(\frac{T_U}{T_M}\right)^{2/3}$$

OR

Substitution of data in equation in any of its forms ✓

The first mark is for converting the equation into a proportion with or without substitution of data.

$$\text{Eg } r_U = 1.29 \times 10^8 \left(\frac{4.14}{1.41} \right)^{2/3}$$

$$(r_U =) \text{ radius} = 2.6(5) \times 10^8 \text{ (m)} \quad \checkmark$$

Last mark is for evaluating the correct answer

Answer only gains both marks

2

(c) Rearranging

$$T^2 = \frac{4\pi^2}{GM} r^3$$

To give

$$M = \frac{4\pi^2}{T^2 G} r^3 \quad \text{or} \quad M = \frac{4\pi^2}{Gk}$$

Or substitution of data $\checkmark$ (allow use of their calculated k from (b))

Conversion of T to seconds for Miranda

$$T = 1.41 \times 24 \times 3600 = 1.22 \times 10^5 \text{ s} \quad \checkmark \quad (\text{or } T^2 = 2.48 \times 10^{10})$$

Substitution into equation and determination of M

$$M = 8.5(6) \times 10^{25} \text{ kg} \quad \checkmark$$

Give full credit for use of Umbriel period and their answer to (b)

Alternative:

Conversion of T to seconds for Umbriel

$$T = 4.14 \times 24 \times 3600 = 3.58 \times 10^5 \text{ s} \quad \checkmark$$

Determine k from

$$T^2 = kr^3 \text{ for Miranda (or Umbriel)} \quad \checkmark$$

$$k = 6.9 \times 10^{-15}$$

Converting to seconds mark stands alone

$$\text{use of } k = \frac{4\pi^2}{GM}$$

$$\text{to find } M = 8.56 \times 10^{25} \text{ kg} \quad \checkmark$$

No ecf for final answer mark.

3

$$(d) \quad \frac{GMm}{(d/2)} = \frac{1}{2}mv^2$$

OR

$$v = \left(\frac{4GM}{d} \right)^{1/2}$$

OR

stating the escape velocity depends on M/d $\checkmark$

Any correct calculation of an escape velocity or ratio M/d

$$\text{or } \left(\frac{M}{d} \right)^{1/2} \quad \checkmark$$

Last mark only given with three correct relevant calculations

Correct conclusion = Titania ✓

Full credit may be given for answers that use r rather than d .

Table to help identify 2nd mark

Name	m/d	(m/d) ^{1/2}	v
Ariel	1.09×10^{15}	3.31×10^7	540
Oberon	1.99×10^{15}	4.46×10^7	729
Titania	2.21×10^{15}	4.70×10^7	768

3

(e) (On Ariel surface $g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \times 1.27 \times 10^{21}}{(1.16 \times 10^6 / 2)^2}$)

$g = 0.25 \text{ (m s}^{-2}\text{)} \checkmark$

(spring supplies the same potential energy so

$(mg h)_{\text{Earth}} = (mg h)_{\text{Ariel}}$

$9.8 \times 1.0 = 0.25 \times h$

OR

(comparing $\frac{v^2 - u^2}{2a}$

$h = \frac{0 - 20}{-2 \times 0.25} \checkmark$

$h = 39 \text{ or } 40 \text{ m}$, so it could not ✓ {allow ecf for arithmetic errors only}

The second mark can be given for the idea that the gravitational potential energy gained on the Earth is the same as that on Ariel with or without data.

Or that the initial kinetic energy is the same in both cases.

Last mark allow ecf for arithmetic errors only.

Condone incorrect use of signs.

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